where

$$P_n P_m = 4\cos\left[\left(\theta - \delta - \frac{\phi}{2}\right)(m-n)\right]\cos\left(\frac{\psi + n\phi}{2}\right)\cos\left(\frac{\psi + m\phi}{2}\right) \tag{42}$$

The function $L_n^{(\gamma)}(y)$ in Eq. (41) is the associated Laguerre polynomial defined as

$$L_n^{(\gamma)}(y) = \sum_{i=0}^{n} (-)^i \frac{(n+\gamma)!\ y^i}{(n-i)!\ (\gamma+i)!\ i!}, \qquad \gamma = 0, 1, 2 \ldots \tag{43}$$

Note that is our calculations we have taken $\eta = |\eta| e^{i\theta}$ and $\alpha = |\alpha| e^{i\delta}$, where θ and δ are the phases of η and α respectively.

The Wigner function of phased orthogonal even binomial state is

$$\begin{aligned} W(\alpha) = {} & \frac{2A_e}{\pi}(1 - |\eta|^2)^M \exp(-2|\alpha|^2) \\ & \left[\sum_{n=0}^{[M/2]} \binom{M}{2n} \left(\frac{|\eta|^2}{1-|\eta|^2}\right)^{2n} P_{2n}^2\, L_{2n}(4|\alpha|^2) \right. \\ & + 2 \sum_{\substack{m,n=0 \\ m>n}}^{[M/2]} \sqrt{\frac{2n!}{2m!} \binom{M}{2n} \binom{M}{2m}} \\ & \left. \left(\frac{|\eta|^2}{1-|\eta|^2}\right)^{m+n} (4|\alpha|^2)^{m-n} P_{2n} P_{2m} L_{2n}^{2(m-n)}(4|\alpha|^2) \right] \end{aligned} \tag{44a}$$

and

$$P_{2n} P_{2m} = 4\cos[(2\theta - 2\delta - \phi)(m-n)]\cos\left(\frac{\psi}{2} + n\phi\right)\cos\left(\frac{\psi}{2} + m\phi\right). \tag{44b}$$

While the Q-function is

$$\begin{aligned} Q(\alpha) = {} & \frac{A_e}{\pi}(1 - |\eta|^2)^M \exp(-|\alpha|^2) \\ & \left[\sum_{n=0}^{[M/2]} \binom{M}{2n} \left(\frac{|\eta|^2}{1-|\eta|^2}\right)^{2n} \frac{(|\alpha|^2)^{2n}}{2n!} P_{2n}^2 \right. \end{aligned}$$

$$+ 2\sum_{\substack{m,n=0\\ m>n}}^{[M/2]} \sqrt{\binom{M}{2n}\binom{M}{2m}} \left(\frac{|\eta|^2}{1-|\eta|^2}\right)^{(m+n)} \frac{|\alpha|^{2(m+n)}}{\sqrt{2n!\,2m!}} P_{2n}\, P_{2m}\Bigg] \tag{45}$$

Finally we would like to mention that; as a result of the non-classical character of the present state, we find the P-representation is highly singular [14]. Therefore the consideration of the P-representation for the state $|\chi>$ is meaningless. Now we shall make use of the Wigner function (44) to get the probability distribution function $P(x)$ by integrating $W(\alpha)$, with $(\alpha = x + iy)$ over the imaginary variable y [14], where

$$P(x) = \int_{-\infty}^{\infty} W(x+iy)dy \tag{46}$$

Substituting Eq. (44) into Eq. (46), and evaluating the integral we get

$$\begin{aligned} P(x) = & \sqrt{\frac{2}{\pi}} A_e (1-|\eta|^2)^M \exp(-2x^2) \\ & \Bigg[\sum_{n=0}^{[M/2]} \binom{M}{2n} \left(\frac{|\eta|^2}{2(1-|\eta|^2)}\right)^{2n} P_{2n}^2 \frac{H_{2n}^2(\sqrt{2}x)}{2n!} \\ & + 2\sum_{\substack{m,n=0\\ m>n}}^{[M/2]} \sqrt{\binom{M}{2n}\binom{M}{2m}} \left(\frac{|\eta|^2}{2(1-|\eta|^2)}\right)^{m+n} \\ & P'_{2n} P'_{2m} \frac{H_{2m}(\sqrt{2}x) H_{2n}(\sqrt{2}x)}{\sqrt{2n!2m!}}\Bigg] \end{aligned} \tag{47a}$$

where $H_m(z)$ is the Hermite polynomial of order m:

$$H_m(z) = \sum_{r=0}^{[m/2]} \frac{(-1)^r m!(2z)^{m-2r}}{r!(m-2r)!} \tag{47b}$$

and

$$P'_{2n} P'_{2m} = 4\cos\left[\left(2\theta - \frac{\pi}{2}\right)(m-n)\right] \cos\left(\frac{\psi}{2} + n\frac{\pi}{2}\right) \cos\left(\frac{\psi}{2} + m\frac{\pi}{2}\right) \tag{48}$$